Reading 3 (Due Wednesday 7/3/24 by 12:55 PM)

Directions: Review the basics of derivatives, antiderivatives, and integrals from single-variable calculus, if necessary. Then read the following sections of the book:

- Section 9.7.1.

and complete the following tasks along the way. If an Activity is not listed, you do not need to complete it (although you are welcome to read it). Turn your write up in via gradescope. You do not need to write the questions down, as long as you clearly indicate the question number.

1. Preview Activity 9.7.1. Before beginning this activity, you should review the derivative of a function $f(x)$. In particular, try to understand why the derivative $f'(x)$ measures the slope of the tangent line to the graph of f at the point $(x, f(x))$. Here is a visualization of what's going on: GeoGebra: Tangent Line Approximation by Secant Line. Notice that the difference quotient $\frac{f(x+h)-f(x)}{h}$ is the slope of the secant line through the points $(x, f(x))$ and $(x+h, f(x+h))$. As we take $h \to 0$ in the limit, the secant line approaches the tangent line. Thus, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ is the slope of the tangent line to the graph of f at the point $(x, f(x))$. We will be generalizing this idea to define the derivative of a vector-valued function.
2. We have seen that a vector-valued function $\mathbf{r}(t)$ that describes a line through a point P in the direction of $\mathbf{v}$ is given by $\mathbf{r}(t) = \overrightarrow{OP} + t\mathbf{v}$. Based on the intuition you developed in Preview Activity 9.7.1, what do you think the derivative $\mathbf{r}'(t)$ is? What is its direction relative to the line traced out by $\mathbf{r}(t)$? What does it represent? Explain your reasoning.
3. Suppose that $\mathbf{s}(t)$ is a vector-valued function that describes the displacement $\mathbf{s}(t)$ of an object with respect to the origin. Based on your intuition and previous studies of derivatives, what you do think the vector $\mathbf{s}'(t_0)$ at time t_0 represents? What is it's direction relative to the curve traced out by $\mathbf{s}(t)$? Explain your reasoning.
4. Activity 9.7.2
5. After completing the preceding tasks, Write down 3 things you learned or still have questions about in Section 9.7.1.

Basic learning objectives: These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated in italics.

1. State the *definition* of and describe geometrically the derivative $\mathbf{r}'(t)$ of a vector-valued function $\mathbf{r}(t)$. Give a physical interpretation when $\mathbf{r}(t)$ describes displacement or velocity.
2. Compute the derivative of a vector-valued function using the *definition*.

Advanced learning objectives: In addition to mastering the basic objectives, here are the tasks you should be able to perform **after class, with sufficient practice:**

1. Compute derivatives of vector-valued functions using both the definition and other techniques. In particular, utilize the differentiation rules to perform more advanced computations.
2. Compute the tangent line to a curve. Compare and contrast the tangent line to a curve with the tangent line to a graph from ordinary calculus. Recognize a tangent line as a linear approximation to a curve.
3. Integrate vector valued functions using antiderivatives. Give a physical interpretation when the vector-valued function you are integrating represents velocity or acceleration.